Announcements

1) Quiz Thursday covering 3.1 & 3.3

2) Exam next Thursday 11/3 covering 1.8 (intermediate value theorem), 2.1 – 2.8, 3.1 – 3.5

Chapter 3.

Derivatives and the Shape of a Graph

Section 3.1 Maximum / Minimum values

Recall: A function f defined on an interval I is said to be **increasing** on I if for all $a, b \in I$ with $a < b$, $f(a) \leq f(b)$.

Similarly, f is **decreasing** on I if for all $a, b \in I$ with $a < b$, $f(a) \geq f(b)$.

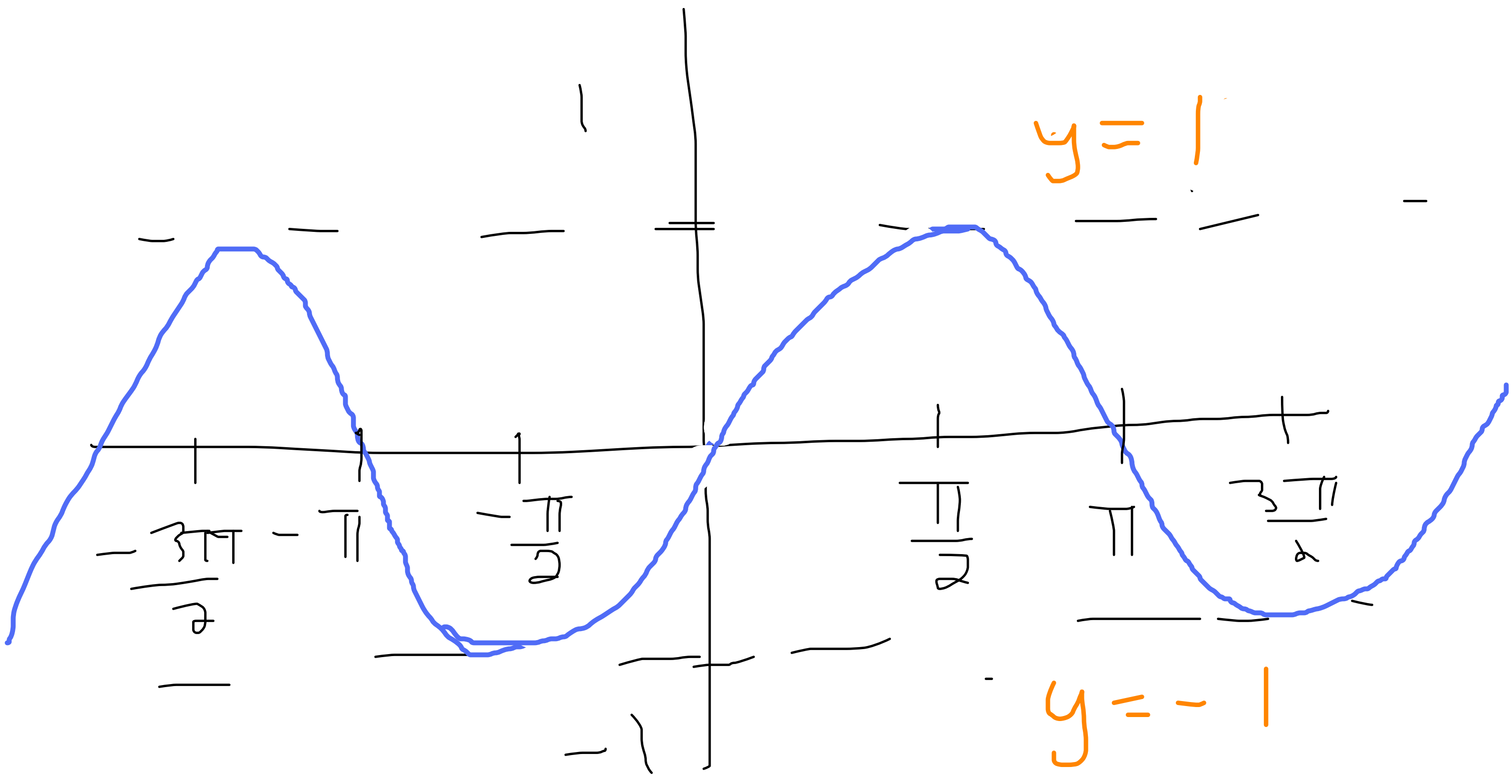
f is **strictly** increasing if $a < b$ implies $f(a) < f(b)$.

f is **strictly** decreasing if $a < b$ implies $f(a) > f(b)$

Example: $f(x) = x$ or $f(x) = x^3$ are strictly increasing on all real numbers.

Example 1. $f(x) = \sin(x)$

Picture



Sine is increasing from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$, decreasing from $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$

Observe that if f is strictly increasing,

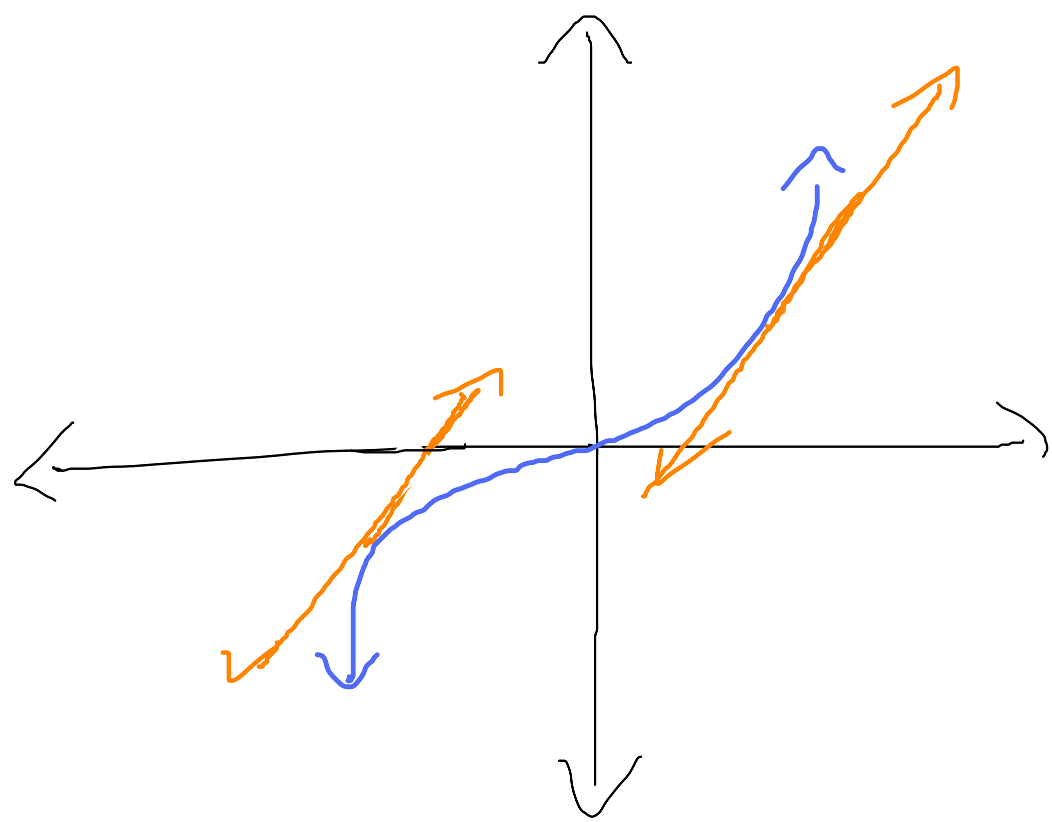
then

$$\frac{f(a)-f(b)}{a-b} = \frac{f(b)-f(a)}{b-a} > 0 \text{ if } a<b.$$

It then stands to reason that if f is strictly increasing on an interval I and c is a point in I,

$$\boxed{f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c} > 0}$$

Imagine $f(x) = x^3$

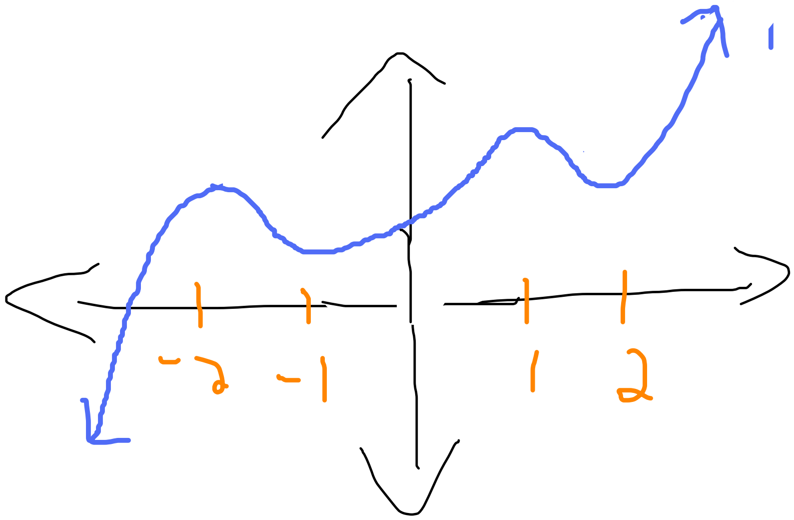


We can see that, except for $x = 0$, the slope of every tangent line is positive.

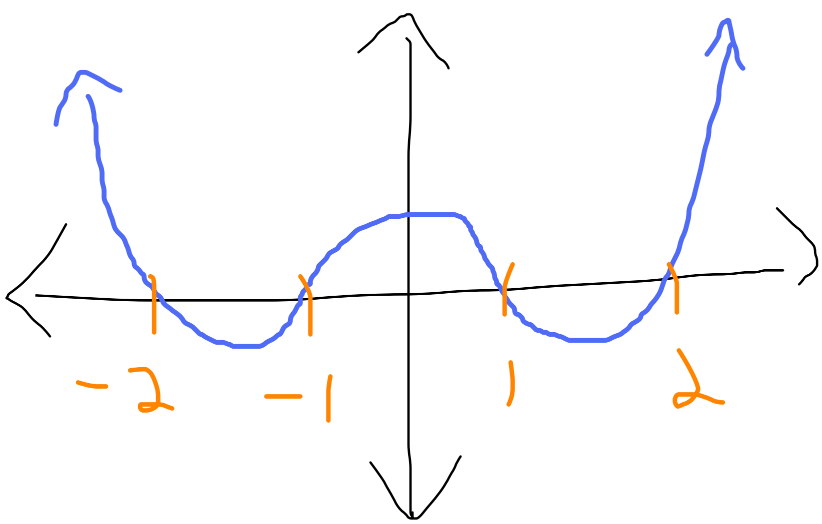
Similarly, if f is strictly decreasing on an interval I, $f'(c)$ should be negative for c in I.

Picture (crude)

If this is f



Then f' looks (roughly) like



In fact, the reverse is true: If $f'(x) > 0$ for all x in an interval I, then f is increasing on I. Similarly, if $f'(x) < 0$ for all x in I, then f is decreasing on I.

To show this. we need . . .

Mean Value Theorem

If f is continuous on $[a,b]$ $(a \neq b)$ and differentiable on (a,b), then there is a point c in (a,b) with

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

Extra Credit

(due Monday 11/7)

1) Show that if f is any quadratic function, then $c = \frac{a+b}{2}$.

2) Show that if f is twice-differentiable and the point c in the mean value theorem is always $\frac{a+b}{2}$, then f is a quadratic function

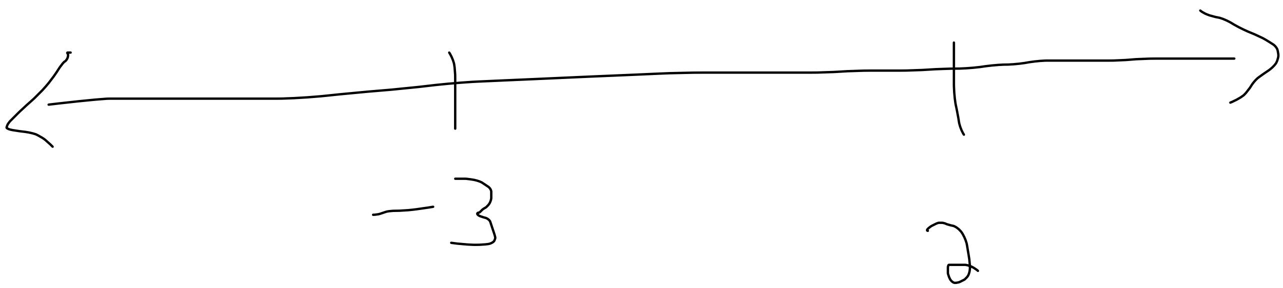
Example 2: Find the intervals where $f(x) = 2x^3 + 3x^2 - 36x + 4$ is increasing / decreasing.

Take derivative, set it equal to zero.

$$f'(x) = 6x^2 + 6x - 36$$
$$= 6(x^2 + x - 6)$$
$$= 6(x+3)(x-2)$$
$$= 0$$

when $x = -3$ or $x = 2$.

Plot these points on a number line.



These points determine 3 open intervals

$(-\infty, -3), (-3, 2), (2, \infty)$.

On each interval, f' is either always positive or always negative.

To find out which, pick a point in the open intervals, plug them back into f'.

/

Points

On $(-3, 2)$ choose $x = 0$

$$f'(0) = -36 < 0$$

so f is decreasing

On $(-\infty, -3)$ choose $x = -4$

$$f'(-4) = 6(-4+3)(-4-2)$$

$$= 6(-1)(-6)$$

$$= 36 > 0$$

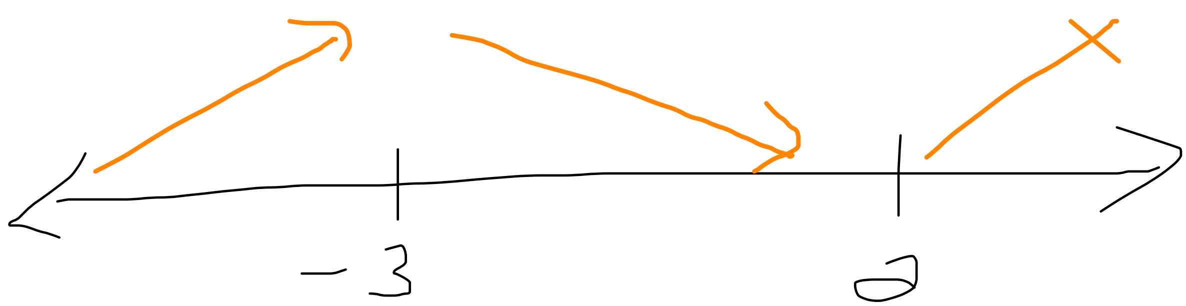
so f is increasing

On $(2, \infty)$ Choose $x = 7$

$$f(7) = 6(7+3)(7-2)$$
$$= 6(10)(5)$$
$$= 360 > 0$$

So f is increasing.

Draw lines w/ positive slope for increasing, negative for decreasing.



f is increasing on $(-\infty, -3)$ and $(2, \infty)$, decreasing on $(-3, 2)$.

Question: What about points where f' is zero or does not exist?

Definition (absolute max/min)

f has an **absolute maximum** at $x=c$ if $f(x) \le f(c)$ for all x.

f has an **absolute minimum** at $x=c$ if $f(x) \ge f(c)$ for all x.